TRIGONOMETRIC EQUATION

1. TRIGONOMETRIC EQUATION :

An equation involving one or more trigonometrical ratios of unknown angles is called a trigonometrical equation.

2. SOLUTION OF TRIGONOMETRIC EQUATION :

A value of the unknown angle which satisfies the given equation is called a solution of the trigonometric equation.

(a) **Principal solution** :- The solution of the trigonometric equation lying in the interval $[0, 2\pi)$.

(b) **General solution** :- Since all the trigonometric functions are many one & periodic, hence there are infinite values of θ for which trigonometric functions have the same value. All such possible values of θ for which the given trigonometric function is satisfied is given by a general formula. Such a general formula is called general solution of trigonometric equation.

(c) **Particular solution** :- The solution of the trigonometric equation lying in the given interval.

3. GENERAL SOLUTIONS OF SOME TRIGONOMETRIC EQUATIONS (TO BE REMEMBERED) :

(a) If $\sin\theta = 0$, then $\theta = n\pi$, $n \in I$ (set of integers)

(b) If $\cos\theta = 0$, then $\theta = (2n+1)\dfrac{\pi}{2}$, $n \in I$

(c) If $\tan\theta = 0$, then $\theta = n\pi$, $n \in I$

(d) If $\sin\theta = \sin\alpha$, then $\theta = n\pi + (-1)^n\alpha$ where $\alpha \in \left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$, $n \in I$

(e) If $\cos\theta = \cos\alpha$, then $\theta = 2n\pi \pm \alpha$, $n \in I$, $\alpha \in [0,\pi]$

(f) If $\tan\theta = \tan\alpha$, then $\theta = n\pi + \alpha$, $n \in I$, $\alpha \in \left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$

(g) If $\sin\theta = 1$, then $\theta = 2n\pi + \dfrac{\pi}{2} = (4n+1)\dfrac{\pi}{2}$, $n \in I$

(h) If $\cos\theta = 1$ then $\theta = 2n\pi$, $n \in I$

(i) If $\sin^2\theta = \sin^2\alpha$ or $\cos^2\theta = \cos^2\alpha$ or $\tan^2\theta = \tan^2\alpha$, then $\theta = n\pi \pm \alpha$, $n \in I$

(j) For $n \in I$, $\sin n\pi = 0$ and $\cos n\pi = (-1)^n$, $n \in I$

$\sin(n\pi + \theta) = (-1)^n \sin\theta$ $\qquad$ $\cos(n\pi + \theta) = (-1)^n \cos\theta$

(k) $\cos n\pi = (-1)^n$, $n \in I$

If n is an odd integer, then $\sin\dfrac{n\pi}{2} = (-1)^{\frac{n-1}{2}}, \cos\dfrac{n\pi}{2} = 0,$

$$\sin\left(\frac{n\pi}{2} + \theta\right) = (-1)^{\frac{n-1}{2}} \cos\theta$$

$$\cos\left(\frac{n\pi}{2} + \theta\right) = (-1)^{\frac{n+1}{2}} \sin\theta$$

Illustration 1 : Find the set of values of x for which $\dfrac{\tan 3x - \tan 2x}{1 + \tan 3x . \tan 2x} = 1$.

Solution : We have, $\dfrac{\tan 3x - \tan 2x}{1 + \tan 3x . \tan 2x} = 1 \Rightarrow \tan(3x - 2x) = 1 \Rightarrow \tan x = 1$

$\Rightarrow \tan x = \tan\dfrac{\pi}{4} \Rightarrow x = n\pi + \dfrac{\pi}{4}$, $n \in I$ $\qquad$ {using $\tan\theta = \tan\alpha \Leftrightarrow \theta = n\pi + \alpha$)

But for this value of x, tan 2x is not defined.

Hence the solution set for x is ϕ. **Ans.**

Do yourself-1 :

(i) Find general solutions of the following equations :

(a) $\sin\theta = \frac{1}{2}$ (b) $\cos\left(\frac{3\theta}{2}\right) = 0$ (c) $\tan\left(\frac{3\theta}{4}\right) = 0$

(d) $\cos^2 2\theta = 1$ (e) $\sqrt{3}\sec 2\theta = 2$ (f) $\operatorname{cosec}\left(\frac{\theta}{2}\right) = -1$

4. IMPORTANT POINTS TO BE REMEMBERED WHILE SOLVING TRIGONOMETRIC EQUATIONS :

(a) For equations of the type $\sin\theta = k$ or $\cos\theta = k$, one must check that $|k| \le 1$.

(b) Avoid squaring the equations, if possible, because it may lead to extraneous solutions. Reject extra solutions if they do not satisfy the given equation.

(c) Do not cancel the common variable factor from the two sides of the equations which are in a product because we may loose some solutions.

(d) The answer should not contain such values of θ, which make any of the terms undefined or infinite.

(i) Check that denominator is not zero at any stage while solving equations.

(ii) If $\tan\theta$ or $\sec\theta$ is involved in the equations, θ should not be odd multiple of $\frac{\pi}{2}$.

(iii) If $\cot\theta$ or $\operatorname{cosec}\theta$ is involved in the equation, θ should not be multiple of π or 0.

5. DIFFERENT STRATEGIES FOR SOLVING TRIGONOMETRIC EQUATIONS :

(a) Solving trigonometric equations by factorisation.

e.g. $(2\sin x - \cos x)(1 + \cos x) = \sin^2 x$

$\therefore$ $(2\sin x - \cos x)(1 + \cos x) - (1 - \cos^2 x) = 0$

$\therefore$ $(1 + \cos x)(2\sin x - \cos x - 1 + \cos x) = 0$

$\therefore$ $(1 + \cos x)(2\sin x - 1) = 0$

$\Rightarrow$ $\cos x = -1$ or $\sin x = \frac{1}{2}$

$\Rightarrow$ $\cos x = -1 = \cos\pi$ $\Rightarrow$ $x = 2n\pi + \pi = (2n + 1)\pi,\ n \in I$

or $\sin x = \frac{1}{2} = \sin\frac{\pi}{6}$ $\Rightarrow$ $x = k\pi + (-1)^k \frac{\pi}{6},\ k \in I$

Illustration 2 : If $\frac{1}{6}\sin\theta$, $\cos\theta$ and $\tan\theta$ are in G.P. then the general solution for θ is -

(A) $2n\pi \pm \frac{\pi}{3}$ (B) $2n\pi \pm \frac{\pi}{6}$ (C) $n\pi \pm \frac{\pi}{3}$ (D) none of these

Solution : Since, $\frac{1}{6}\sin\theta$, $\cos\theta$, $\tan\theta$ are in G.P.

$\Rightarrow$ $\cos^2\theta = \frac{1}{6}\sin\theta \cdot \tan\theta$ $\Rightarrow$ $6\cos^3\theta + \cos^2\theta - 1 = 0$

$\therefore$ $(2\cos\theta - 1)(3\cos^2\theta + 2\cos\theta + 1) = 0$

$\Rightarrow$ $\cos\theta = \frac{1}{2}$ (other values of $\cos\theta$ are imaginary)

$\Rightarrow$ $\cos\theta = \cos\frac{\pi}{3}$ $\Rightarrow$ $\theta = 2n\pi \pm \frac{\pi}{3},\ n \in I.$ **Ans. (A)**

(b) **Solving of trigonometric equation by reducing it to a quadratic equation.**

e.g. $6 - 10\cos x = 3\sin^2 x$

$\therefore \quad 6 - 10\cos x = 3 - 3\cos^2 x \quad \Rightarrow \quad 3\cos^2 x - 10\cos x + 3 = 0$

$\Rightarrow \quad (3\cos x - 1)(\cos x - 3) = 0 \quad \Rightarrow \quad \cos x = \frac{1}{3}$ or $\cos x = 3$

Since $\cos x = 3$ is not possible as $-1 \le \cos x \le 1$

$\therefore \quad \cos x = \frac{1}{3} = \cos\left(\cos^{-1}\frac{1}{3}\right) \quad \Rightarrow \quad x = 2n\pi \pm \cos^{-1}\left(\frac{1}{3}\right),\ n \in I$

Illustration 3 : Solve $\sin^2\theta - \cos\theta = \frac{1}{4}$ for θ and write the values of θ in the interval $0 \le \theta \le 2\pi$.

Solution : The given equation can be written as

$1 - \cos^2\theta - \cos\theta = \frac{1}{4} \quad \Rightarrow \quad \cos^2\theta + \cos\theta - 3/4 = 0$

$\Rightarrow \quad 4\cos^2\theta + 4\cos\theta - 3 = 0 \quad \Rightarrow \quad (2\cos\theta - 1)(2\cos\theta + 3) = 0$

$\Rightarrow \quad \cos\theta = \frac{1}{2}, -\frac{3}{2}$

Since, $\cos\theta = -3/2$ is not possible as $-1 \le \cos\theta \le 1$

$\therefore \quad \cos\theta = \frac{1}{2} \quad \Rightarrow \quad \cos\theta = \cos\frac{\pi}{3} \quad \Rightarrow \quad \theta = 2n\pi \pm \frac{\pi}{3},\ n \in I$

For the given interval, $n = 0$ and $n = 1$.

$\Rightarrow \quad \theta = \frac{\pi}{3}, \frac{5\pi}{3}$ **Ans.**

Illustration 4 : Find the number of solutions of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi]$.

Solution : Here, $\tan x + \sec x = 2\cos x \quad \Rightarrow \quad \sin x + 1 = 2\cos^2 x$

$\Rightarrow \quad 2\sin^2 x + \sin x - 1 = 0 \quad \Rightarrow \quad \sin x = \frac{1}{2}, -1$

But $\sin x = -1 \Rightarrow x = \frac{3\pi}{2}$ for which $\tan x + \sec x = 2\cos x$ is not defined.

Thus $\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\Rightarrow$ number of solutions of $\tan x + \sec x = 2\cos x$ is 2. **Ans.**

Illustration 5 : Solve the equation $5\sin^2 x - 7\sin x\cos x + 16\cos^2 x = 4$

Solution : To solve this equation we use the fundamental formula of trigonometric identities,

$\sin^2 x + \cos^2 x = 1$

writing the equation in the form,

$5\sin^2 x - 7\sin x \cdot \cos x + 16\cos^2 x = 4(\sin^2 x + \cos^2 x)$

$\Rightarrow \quad \sin^2 x - 7\sin x\cos x + 12\cos^2 x = 0$

dividing by $\cos^2 x$ on both side we get,

$\tan^2 x - 7\tan x + 12 = 0$

Now it can be factorized as :

$(\tan x - 3)(\tan x - 4) = 0$

$\Rightarrow \quad \tan x = 3, 4$

i.e., $\tan x = \tan(\tan^{-1}3)$ or $\tan x = \tan(\tan^{-1}4)$

$\Rightarrow \quad x = n\pi + \tan^{-1}3$ or $x = n\pi + \tan^{-1}4,\ n \in I.$ **Ans.**

Illustration 6 : If $x \neq \frac{n\pi}{2}$, $n \in I$ and $(\cos x)^{\sin^2 x - 3\sin x + 2} = 1$, then find the general solutions of x.

Solution : As $x \neq \frac{n\pi}{2} \Rightarrow \cos x \neq 0, 1, -1$

So, $(\cos x)^{\sin^2 x - 3\sin x + 2} = 1 \Rightarrow \sin^2 x - 3\sin x + 2 = 0$

$\therefore$ $(\sin x - 2)(\sin x - 1) = 0 \Rightarrow \sin x = 1, 2$

where sinx = 2 is not possible and sinx = 1 which is also not possible as $x \neq \frac{n\pi}{2}$

$\therefore$ no general solution is possible. **Ans.**

Illustration 7 : Solve the equation $\sin^4 x + \cos^4 x = \frac{7}{2}\sin x \cdot \cos x$.

Solution : $\sin^4 x + \cos^4 x = \frac{7}{2}\sin x \cdot \cos x \Rightarrow (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x = \frac{7}{2}\sin x \cdot \cos x$

$\Rightarrow 1 - \frac{1}{2}(\sin 2x)^2 = \frac{7}{4}(\sin 2x) \Rightarrow 2\sin^2 2x + 7\sin 2x - 4 = 0$

$\Rightarrow (2\sin 2x - 1)(\sin 2x + 4) = 0 \Rightarrow \sin 2x = \frac{1}{2}$ or $\sin 2x = -4$ (which is not possible)

$\Rightarrow 2x = n\pi + (-1)^n\frac{\pi}{6}$, $n \in I$

i.e., $x = \frac{n\pi}{2} + (-1)^n\frac{\pi}{12}$, $n \in I$ **Ans.**

Do yourself-2 :

(i) Solve the following equations :

(a) $3\sin x + 2\cos^2 x = 0$ (b) $\sec^2 2\alpha = 1 - \tan 2\alpha$

(c) $7\cos^2\theta + 3\sin^2\theta = 4$ (d) $4\cos\theta - 3\sec\theta = \tan\theta$

(ii) Solve the equation : $2\sin^2\theta + \sin^2 2\theta = 2$ for $\theta \in (-\pi, \pi)$.

(c) Solving trigonometric equations by introducing an auxilliary argument.

Consider, $a\sin\theta + b\cos\theta = c$ .... (i)

$$\therefore \quad \frac{a}{\sqrt{a^2+b^2}}\sin\theta + \frac{b}{\sqrt{a^2+b^2}}\cos\theta = \frac{c}{\sqrt{a^2+b^2}}$$

equation (i) has a solution only if $|c| \leq \sqrt{a^2+b^2}$

let $\frac{a}{\sqrt{a^2+b^2}} = \cos\phi$, $\frac{b}{\sqrt{a^2+b^2}} = \sin\phi$ & $\phi = \tan^{-1}\frac{b}{a}$

by introducing this auxillary argument ϕ, equation (i) reduces to

$\sin(\theta + \phi) = \frac{c}{\sqrt{a^2+b^2}}$ Now this equation can be solved easily.

Illustration 8 : Find the number of distinct solutions of $\sec x + \tan x = \sqrt{3}$, where $0 \leq x \leq 3\pi$.

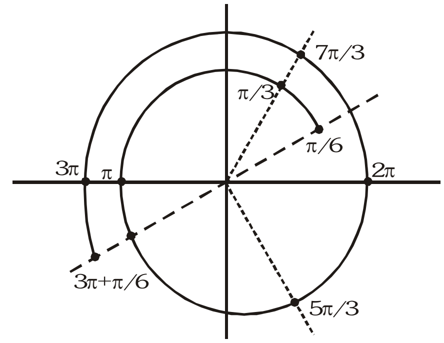
Solution : Here, $\sec x + \tan x = \sqrt{3} \Rightarrow 1 + \sin x = \sqrt{3}\cos x$

or $\sqrt{3}\cos x - \sin x = 1$

dividing both sides by $\sqrt{a^2+b^2}$ i.e. $\sqrt{4} = 2$, we get

$\Rightarrow \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = \frac{1}{2}$

$\Rightarrow \quad \cos\frac{\pi}{6}\cos x - \sin\frac{\pi}{6}\sin x = \frac{1}{2} \Rightarrow \quad \cos\left(x+\frac{\pi}{6}\right) = \frac{1}{2}$



As $\quad 0 \le x \le 3\pi$

$\frac{\pi}{6} \le x + \frac{\pi}{6} \le 3\pi + \frac{\pi}{6}$

$\Rightarrow \quad x + \frac{\pi}{6} = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3} \qquad \Rightarrow \qquad x = \frac{\pi}{6}, \frac{3\pi}{2}, \frac{13\pi}{6}$

But at $x = \frac{3\pi}{2}$, tanx and secx is not defined.

$\therefore$ Total number of solutions are 2. **Ans.**

Illustration 9 : Prove that the equation kcosx − 3sinx = k + 1 possess a solution iff $k \in (-\infty, 4]$.

Solution : Here, k cosx − 3sinx = k + 1, could be re-written as :

$$\frac{k}{\sqrt{k^2+9}}\cos x - \frac{3}{\sqrt{k^2+9}}\sin x = \frac{k+1}{\sqrt{k^2+9}}$$

or $\quad \cos(x+\phi) = \frac{k+1}{\sqrt{k^2+9}}$, where $\tan\phi = \frac{3}{k}$

which possess a solution only if $-1 \le \frac{k+1}{\sqrt{k^2+9}} \le 1$

i.e., $\quad \left|\frac{k+1}{\sqrt{k^2+9}}\right| \le 1$

i.e., $\quad (k+1)^2 \le k^2 + 9$

i.e., $\quad k^2 + 2k + 1 \le k^2 + 9$

or $\quad k \le 4$

$\Rightarrow$ The interval of k for which the equation (kcosx − 3sinx = k + 1) has a solution is $(-\infty, 4]$. **Ans.**

Do yourself-3 :

(i) Solve the following equations :

(a) $\sin x + \sqrt{2} = \cos x$.

(b) $\operatorname{cosec}\theta = 1 + \cot\theta$.

(d) Solving trigonometric equations by transforming sum of trigonometric functions into product.

e.g. $\quad \cos 3x + \sin 2x - \sin 4x = 0$

$\cos 3x - 2\sin x \cos 3x = 0$

$\Rightarrow \quad (\cos 3x)(1 - 2\sin x) = 0$

$\Rightarrow \quad \cos 3x = 0 \quad$ or $\quad \sin x = \frac{1}{2}$

$\Rightarrow \quad \cos 3x = 0 = \cos\frac{\pi}{2} \quad$ or $\quad \sin x = \frac{1}{2} = \sin\frac{\pi}{6}$

$\Rightarrow \quad 3x = 2n\pi \pm \frac{\pi}{2} \quad$ or $\quad x = m\pi + (-1)^m \frac{\pi}{6}$

$\Rightarrow \quad x = \frac{2n\pi}{3} \pm \frac{\pi}{6} \quad$ or $\quad x = m\pi + (-1)^m \frac{\pi}{6}$; $(n, m \in I)$

Illustration 10 : Solve : $\cos\theta + \cos3\theta + \cos5\theta + \cos7\theta = 0$

Solution : We have $\cos\theta + \cos7\theta + \cos3\theta + \cos5\theta = 0$

$\Rightarrow$ $2\cos4\theta\cos3\theta + 2\cos4\theta\cos\theta = 0$ $\Rightarrow$ $\cos4\theta(\cos3\theta + \cos\theta) = 0$

$\Rightarrow$ $\cos4\theta(2\cos2\theta\cos\theta) = 0$

$\Rightarrow$ Either $\cos\theta = 0 \Rightarrow \theta = (2n_1 + 1)\pi/2,\ n_1 \in I$

or $\cos2\theta = 0 \Rightarrow \theta = (2n_2 + 1)\dfrac{\pi}{4},\ n_2 \in I$

or $\cos4\theta = 0 \Rightarrow \theta = (2n_3 + 1)\dfrac{\pi}{8},\ n_3 \in I$ **Ans.**

(e) **Solving trigonometric equations by transforming a product into sum.**

e.g. $\sin5x.\ \cos3x = \sin6x.\ \cos2x$

$\sin8x + \sin2x = \sin8x + \sin4x$

$\therefore$ $2\sin2x\ .\ \cos2x - \sin2x = 0$

$\Rightarrow$ $\sin2x(2\cos2x - 1) = 0$

$\Rightarrow$ $\sin2x = 0$ or $\cos2x = \dfrac{1}{2}$

$\Rightarrow$ $\sin2x = 0 = \sin0$ or $\cos2x = \dfrac{1}{2} = \cos\dfrac{\pi}{3}$

$\Rightarrow$ $2x = n\pi + (-1)^n\ 0,\ n \in I$ or $2x = 2m\pi \pm \dfrac{\pi}{3},\ m \in I$

$\Rightarrow$ $x = \dfrac{n\pi}{2},\ n \in I$ or $x = m\pi \pm \dfrac{\pi}{6},\ m \in I$

Illustration 11 : Solve : $\cos\theta\cos2\theta\cos3\theta = \dfrac{1}{4}$; where $0 \le \theta \le \pi$.

Solution : $\dfrac{1}{2}(2\cos\theta\cos3\theta)\cos2\theta = \dfrac{1}{4}$ $\Rightarrow$ $(\cos2\theta + \cos4\theta)\cos2\theta = \dfrac{1}{2}$

$\Rightarrow$ $\dfrac{1}{2}[2\cos^2 2\theta + 2\cos4\theta\cos2\theta] = \dfrac{1}{2}$ $\Rightarrow$ $1 + \cos4\theta + 2\cos4\theta\cos2\theta = 1$

$\therefore$ $\cos4\theta\ (1 + 2\cos2\theta) = 0$

$\cos4\theta = 0$ or $(1 + 2\cos2\theta) = 0$

Now from the first equation : $2\cos4\theta = 0 = \cos(\pi/2)$

$\therefore$ $4\theta = \left(n + \dfrac{1}{2}\right)\pi$ $\Rightarrow$ $\theta = (2n+1)\dfrac{\pi}{8},\ n \in I$

for $n = 0,\ \theta = \dfrac{\pi}{8}$; $n = 1,\ \theta = \dfrac{3\pi}{8}$; $n = 2,\ \theta = \dfrac{5\pi}{8}$; $n = 3,\ \theta = \dfrac{7\pi}{8}$ $(\because\ 0 \le \theta \le \pi)$

and from the second equation :

$\cos2\theta = -\dfrac{1}{2} = -\cos(\pi/3) = \cos(\pi-\pi/3) = \cos(2\pi/3)$

$\therefore$ $2\theta = 2k\pi \pm 2\pi/3$ $\therefore$ $\theta = k\pi \pm \pi/3,\ k \in I$

again for $k = 0,\ \theta = \dfrac{\pi}{3}$; $k = 1,\ \theta = \dfrac{2\pi}{3}$ $(\because\ 0 \le \theta \le \pi)$

$\therefore$ $\theta = \dfrac{\pi}{8}, \dfrac{\pi}{3}, \dfrac{3\pi}{8}, \dfrac{5\pi}{8}, \dfrac{2\pi}{3}, \dfrac{7\pi}{8}$ **Ans.**

Do yourself-4 :

(i) Solve $4\sin\theta \sin2\theta \sin4\theta = \sin3\theta$.

(ii) Solve for x : $\sin x + \sin3x + \sin5x = 0$.

(f) Solving equations by a change of variable :

(i) Equations of the form P (sin x ± cos x, sin x. cos x) = 0, where P (y,z) is a polynomial, can be solved by the substitution :

$\cos x \pm \sin x = t \quad \Rightarrow \quad 1 \pm 2\sin x.\cos x = t^2$.

e.g. $\sin x + \cos x = 1 + \sin x.\cos x$.

put $\sin x + \cos x = t$

$\Rightarrow \sin^2x + \cos^2x + 2\sin x.\cos x = t^2$

$\Rightarrow 2\sin x\cos x = t^2 - 1 \quad (\because \sin^2x + \cos^2x = 1)$

$$\Rightarrow \sin x.\cos x = \left(\frac{t^2-1}{2}\right)$$

Substituting above result in given equation, we get :

$$t = 1 + \frac{t^2-1}{2}$$

$\Rightarrow 2t = t^2 + 1 \Rightarrow t^2 - 2t + 1 = 0$

$\Rightarrow (t-1)^2 = 0 \quad \Rightarrow \quad t = 1$

$\Rightarrow \sin x + \cos x = 1$

Dividing both sides by $\sqrt{1^2+1^2}$ i.e. $\sqrt{2}$, we get

$$\Rightarrow \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \cos x\cos\frac{\pi}{4} + \sin x.\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\left(x-\frac{\pi}{4}\right) = \cos\frac{\pi}{4} \quad \Rightarrow \quad x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$$

$$\Rightarrow x = 2n\pi \text{ or } x = 2n\pi + \frac{\pi}{2} = (4n+1)\frac{\pi}{2},\ n \in I$$

(ii) Equations of the form of $a\sin x + b\cos x + d = 0$, where a, b & d are real numbers can be solved by changing sin x & cos x into their corresponding tangent of half the angle.

e.g. $3\cos x + 4\sin x = 5$

$$\Rightarrow 3\left(\frac{1-\tan^2 x/2}{1+\tan^2 x/2}\right) + 4\left(\frac{2\tan x/2}{1+\tan^2 x/2}\right) = 5$$

$$\Rightarrow \frac{3-3\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{8\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = 5$$

$$\Rightarrow 3 - 3\tan^2\frac{x}{2} + 8\tan\frac{x}{2} = 5 + 5\tan^2\frac{x}{2} \quad \Rightarrow \quad 8\tan^2\frac{x}{2} - 8\tan\frac{x}{2} + 2 = 0$$

$$\Rightarrow 4\tan^2\frac{x}{2} - 4\tan\frac{x}{2} + 1 = 0 \quad \Rightarrow \quad \left(2\tan\frac{x}{2} - 1\right)^2 = 0$$

$$\Rightarrow 2\tan\frac{x}{2} - 1 = 0 \Rightarrow \tan\frac{x}{2} = \frac{1}{2} = \tan\left(\tan^{-1}\frac{1}{2}\right)$$

$$\Rightarrow \frac{x}{2} = n\pi + \tan^{-1}\left(\frac{1}{2}\right),\ n \in I \quad \Rightarrow \quad x = 2n\pi + 2\tan^{-1}\frac{1}{2},\ n \in I$$

(iii) Many equations can be solved by introducing a new variable.

e.g. $\sin^4 2x + \cos^4 2x = \sin 2x.\cos 2x$

substituting $\sin 2x.\cos 2x = y$ $\because$ $(\sin^2 2x + \cos^2 2x)^2 = \sin^4 2x + \cos^4 2x + 2\sin^2 2x.\cos^2 2x$

$\Rightarrow$ $\sin^4 2x + \cos^4 2x = 1 - 2\sin^2 2x.\cos^2 2x$ substituting above result in given equation :

$1 - 2y^2 = y$

$\Rightarrow$ $2y^2 + y - 1 = 0$ $\Rightarrow$ $2(y+1)\left(y-\frac{1}{2}\right) = 0$

$\Rightarrow$ $y = -1$ or $y = \frac{1}{2}$ $\Rightarrow$ $\sin 2x.\cos 2x = -1$ or $\sin 2x.\cos 2x = \frac{1}{2}$

$\Rightarrow$ $2\sin 2x.\cos 2x = -2$ or $2\sin 2x.\cos 2x = 1$

$\Rightarrow$ $\sin 4x = -2$ (which is not possible) or $2\sin 2x.\cos 2x = 1$

$\Rightarrow$ $\sin 4x = 1 = \sin \frac{\pi}{2}$ $\Rightarrow$ $4x = n\pi + (-1)^n \frac{\pi}{2}$, $n \in I$ $\Rightarrow x = \frac{n\pi}{4} + (-1)^n \frac{\pi}{8}$, $n \in I$

Illustration 12 : Find the general solution of equation $\sin^4 x + \cos^4 x = \sin x \cos x$.

Solution : Using half-angle formulae, we can represent given equation in the form :

$$\left(\frac{1-\cos 2x}{2}\right)^2 + \left(\frac{1+\cos 2x}{2}\right)^2 = \sin x \cos x$$

$\Rightarrow$ $(1 - \cos 2x)^2 + (1 + \cos 2x)^2 = 4\sin x \cos x$

$\Rightarrow$ $2(1 + \cos^2 2x) = 2\sin 2x$ $\Rightarrow$ $1 + 1 - \sin^2 2x = \sin 2x$

$\Rightarrow$ $\sin^2 2x + \sin 2x = 2$

$\Rightarrow$ $\sin 2x = 1$ or $\sin 2x = -2$ (which is not possible)

$\Rightarrow$ $2x = 2n\pi + \frac{\pi}{2}$, $n \in I$

$\Rightarrow$ $x = n\pi + \frac{\pi}{4}, n \in I$ **Ans.**

(g) Solving trigonometric equations with the use of the boundness of the functions involved.

e.g. $\sin x\left(\cos\frac{x}{4} - 2\sin x\right) + \left(1 + \sin\frac{x}{4} - 2\cos x\right).\cos x = 0$

$\therefore$ $\sin x \cos \frac{x}{4} + \cos x \sin \frac{x}{4} + \cos x = 2$

$\therefore$ $\sin\left(\frac{5x}{4}\right) + \cos x = 2$

$\Rightarrow$ $\sin\left(\frac{5x}{4}\right) = 1$ & $\cos x = 1$ (as $\sin\theta \le 1$ & $\cos\theta \le 1$)

Now consider

$\cos x = 1$ $\Rightarrow$ $x = 2\pi, 4\pi, 6\pi, 8\pi$ ....

and $\sin\frac{5x}{4} = 1$ $\Rightarrow$ $x = \frac{2\pi}{5}, \frac{10\pi}{5}, \frac{18\pi}{5}$ ....

Common solution to above APs will be the AP having

First term $= 2\pi$

Common difference = LCM of 2π and $\frac{8\pi}{5} = \frac{40\pi}{5} = 8\pi$

$\therefore$ General solution will be general term of this AP i.e. $2\pi + (8\pi)n,\ n \in I$

$\Rightarrow$ $x = 2(4n + 1)\pi,\ n \in I$

Illustration 13 : Solve the equation $(\sin x + \cos x)^{1+\sin 2x} = 2$, when $0 \le x \le \pi$.

Solution : We know, $-\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$ and $-1 \le \sin\theta \le 1$.

$\therefore$ $(\sin x + \cos x)$ admits the maximum value as $\sqrt{2}$

and $(1 + \sin 2x)$ admits the maximum value as 2.

Also $\left(\sqrt{2}\right)^2 = 2$.

$\therefore$ the equation could hold only when, $\sin x + \cos x = \sqrt{2}$ and $1 + \sin 2x = 2$

Now, $\sin x + \cos x = \sqrt{2}$ $\Rightarrow$ $\cos\left(x - \frac{\pi}{4}\right) = 1$

$\Rightarrow$ $x = 2n\pi + \pi/4,\ n \in I$ ...... (i)

and $1 + \sin 2x = 2$ $\Rightarrow$ $\sin 2x = 1 = \sin\frac{\pi}{2}$

$\Rightarrow$ $2x = m\pi + (-1)^m\frac{\pi}{2},\ m \in I$ $\Rightarrow$ $x = \frac{m\pi}{2} + (-1)^m\frac{\pi}{4}$ ...... (ii)

The value of x in $[0, \pi]$ satisfying equations (i) and (ii) is $x = \frac{\pi}{4}$ (when n = 0 & m = 0) **Ans.**

Note : $\sin x + \cos x = -\sqrt{2}$ and $1 + \sin 2x = 2$ also satisfies but as $x \geq 0$, this solution is not in domain.

Illustration 14 : Solve for x and y : $2^{\frac{1}{\cos^2 x}}\sqrt{y^2 - y + 1/2} \le 1$

Solution :

$$2^{\frac{1}{\cos^2 x}}\sqrt{y^2 - y + 1/2} \le 1 \quad .... \text{(i)}$$

$$2^{\frac{1}{\cos^2 x}}\sqrt{\left(y - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \le 1$$

Minimum value of $2^{\frac{1}{\cos^2 x}} = 2$

Minimum value of $\sqrt{\left(y - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{2}$

$\Rightarrow$ Minimum value of $2^{\frac{1}{\cos^2 x}}\sqrt{y^2 - y + \frac{1}{2}}$ is 1

$\Rightarrow$ (i) is possible when $2^{\frac{1}{\cos^2 x}}\sqrt{\left(y - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = 1$

$\Rightarrow$ $\cos^2 x = 1$ and $y = 1/2$ $\Rightarrow \cos x = \pm 1 \Rightarrow$ $x = n\pi$, where $n \in I$.

Hence $x = n\pi,\ n \in I$ and $y = 1/2$. **Ans.**

Illustration 15 : The number of solution(s) of $2\cos^2\left(\frac{x}{2}\right)\sin^2x = x^2+\frac{1}{x^2}$, $0 \le x \le \pi/2$, is/are -

(A) 0 (B) 1 (C) infinite (D) none of these

Solution : Let $y = 2\cos^2\left(\frac{x}{2}\right)\sin^2x = x^2+\frac{1}{x^2}$ $\Rightarrow$ $y = (1 + \cos x)\sin^2x$ and $y = x^2 + \frac{1}{x^2}$

when $y = (1 + \cos x)\sin^2x$ = (a number < 2)(a number $\le$ 1) $\Rightarrow$ $y < 2$ .... (i)

and when $y = x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2 \ge 2$ $\Rightarrow$ $y \ge 2$ .... (ii)

No value of y can be obtained satisfying (i) and (ii), simultaneously

$\Rightarrow$ No real solution of the equation exists. **Ans. (A)**

Note: If L.H.S. of the given trigonometric equation is always less than or equal to k and RHS is always greater than k, then no solution exists. If both the sides are equal to k for same value of θ, then solution exists and if they are equal for different values of θ, then solution does not exist.

Do yourself-5 :

(i) If $x^2 - 4x + 5 - \sin y = 0$, $y \in [0, 2\pi)$, then -

(A) $x = 1, y = 0$ (B) $x = 1, y = \pi/2$ (C) $x = 2, y = 0$ (D) $x = 2, y = \pi/2$

(ii) If $\sin x + \cos x = \sqrt{y+\frac{1}{y}}$, $y > 0$, $x \in [0, \pi]$, then find the least positive value of x satisfying the given condition.

6. TRIGONOMETRIC INEQUALITIES :

There is no general rule to solve trigonometric inequations and the same rules of algebra are valid provided the domain and range of trigonometric functions should be kept in mind.

Illustration 16 : Find the solution set of inequality $\sin x > 1/2$.

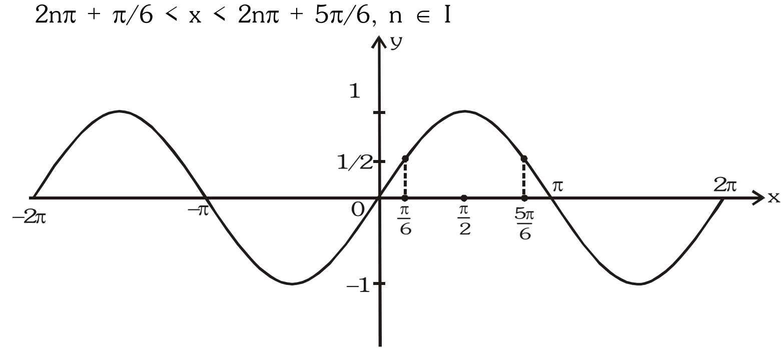
Solution : When $\sin x = \frac{1}{2}$, the two values of x between 0 and 2π are $\pi/6$ and $5\pi/6$.

From the graph of $y = \sin x$, it is obvious that between 0 and 2π,

$\sin x > \frac{1}{2}$ for $\pi/6 < x < 5\pi/6$

Hence, $\sin x > 1/2$

$\Rightarrow$ $2n\pi + \pi/6 < x < 2n\pi + 5\pi/6$, $n \in I$



Thus, the required solution set is $\bigcup_{n\in I}\left(2n\pi+\frac{\pi}{6}, 2n\pi+\frac{5\pi}{6}\right)$ **Ans.**

Illustration 17 : Find the value of x in the interval $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ for which $\sqrt{2}\sin 2x + 1 \le 2\sin x + \sqrt{2}\cos x$

Solution : We have, $\sqrt{2}\sin 2x + 1 \le 2\sin x + \sqrt{2}\cos x \Rightarrow 2\sqrt{2}\sin x\cos x - 2\sin x - \sqrt{2}\cos x + 1 \le 0$

$\Rightarrow \quad 2\sin x(\sqrt{2}\cos x - 1) - 1(\sqrt{2}\cos x - 1) \le 0 \qquad \Rightarrow \quad (2\sin x - 1)(\sqrt{2}\cos x - 1) \le 0$

$\Rightarrow \quad \left(\sin x - \frac{1}{2}\right)\left(\cos x - \frac{1}{\sqrt{2}}\right) \le 0$

Above inequality holds when :

Case-I : $\sin x - \frac{1}{2} \le 0$ and $\cos x - \frac{1}{\sqrt{2}} \ge 0 \Rightarrow \sin x \le \frac{1}{2}$ and $\cos x \ge \frac{1}{\sqrt{2}}$

Now considering the given interval of x :

for $\sin x \le \frac{1}{2}$: $x \in \left[-\frac{\pi}{2}, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \frac{3\pi}{2}\right]$ and for $\cos x \ge \frac{1}{\sqrt{2}}$: $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

For both to simultaneously hold true : $x \in \left[-\frac{\pi}{4}, \frac{\pi}{6}\right]$

Case-II : $\sin x - \frac{1}{2} \ge 0$ and $\cos x \le \frac{1}{\sqrt{2}}$

Again, for the given interval of x :

for $\sin x \ge \frac{1}{2}$: $x \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$ and for $\cos x \le \frac{1}{\sqrt{2}}$: $x \in \left[-\frac{\pi}{2}, -\frac{\pi}{4}\right] \cup \left[\frac{\pi}{4}, \frac{3\pi}{2}\right]$

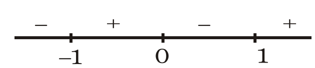
For both to simultaneously hold true : $x \in \left[\frac{\pi}{4}, \frac{5\pi}{6}\right]$

$\therefore$ Given inequality holds for $x \in \left[-\frac{\pi}{4}, \frac{\pi}{6}\right] \cup \left[\frac{\pi}{4}, \frac{5\pi}{6}\right]$ **Ans.**

Illustration 18 : Find the values of α lying between 0 and π for which the inequality : $\tan\alpha > \tan^3\alpha$ is valid.

Solution : We have : $\tan\alpha - \tan^3\alpha > 0 \Rightarrow \tan\alpha\,(1 - \tan^2\alpha) > 0$

$\Rightarrow \quad (\tan\alpha)(\tan\alpha + 1)(\tan\alpha - 1) < 0$



So $\tan\alpha < -1,\ 0 < \tan\alpha < 1$

$\therefore$ Given inequality holds for $\alpha \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$ **Ans.**

Do yourself - 6 :

(i) Find the solution set of the inequality : $\cos x \ge -1/2$.

(ii) Find the values of x in the interval $[0, 2\pi]$ for which $4\sin^2 x - 8\sin x + 3 \le 0$.

__Miscellaneous Illustration :__

Illustration 19 : Solve the following equation : $\tan^2\theta + \sec^2\theta + 3 = 2(\sqrt{2}\sec\theta + \tan\theta)$

Solution : We have $\tan^2\theta + \sec^2\theta + 3 = 2\sqrt{2}\sec\theta + 2\tan\theta$

$\Rightarrow \quad \tan^2\theta - 2\tan\theta + \sec^2\theta - 2\sqrt{2}\sec\theta + 3 = 0$

$\Rightarrow \quad \tan^2\theta + 1 - 2\tan\theta + \sec^2\theta - 2\sqrt{2}\sec\theta + 2 = 0$

$\Rightarrow \quad (\tan\theta - 1)^2 + (\sec\theta - \sqrt{2})^2 = 0 \qquad \Rightarrow \quad \tan\theta = 1$ and $\sec\theta = \sqrt{2}$

As the periodicity of $\tan\theta$ and $\sec\theta$ are not same, we get

$\theta = 2n\pi + \frac{\pi}{4},\ n \in I$ **Ans.**

Illustration 20 : Find the solution set of equation $5^{(1 + \log_5 \cos x)} = 5/2$.

Solution : Taking log to base 5 on both sides in given equation :

$(1 + \log_5 \cos x).\ \log_5 5 = \log_5(5/2) \quad \Rightarrow \log_5 5 + \log_5 \cos x = \log_5 5 - \log_5 2$

$\Rightarrow \log_5 \cos x = -\log_5 2 \Rightarrow \cos x = 1/2 \quad \Rightarrow x = 2n\pi \pm \pi/3,\ n \in I$ **Ans.**

Illustration 21 : If the set of all values of x in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying $|4\sin x + \sqrt{2}| < \sqrt{6}$ is $\left(\frac{a\pi}{24}, \frac{b\pi}{24}\right)$ then find the value of $\left|\frac{a-b}{3}\right|$.

Solution : $|4\sin x + \sqrt{2}| < \sqrt{6}$

$\Rightarrow \quad -\sqrt{6} < 4\sin x + \sqrt{2} < \sqrt{6} \quad \Rightarrow \quad -\sqrt{6} - \sqrt{2} < 4\sin x < \sqrt{6} - \sqrt{2}$

$\Rightarrow \quad \frac{-(\sqrt{6}+\sqrt{2})}{4} < \sin x < \frac{\sqrt{6}-\sqrt{2}}{4} \quad \Rightarrow \quad -\frac{5\pi}{12} < x < \frac{\pi}{12}$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Comparing with $\frac{a\pi}{24} < x < \frac{b\pi}{24}$, we get, $a = -10,\ b = 2$

$\therefore \left|\frac{a-b}{3}\right| = \left|\frac{-10-2}{3}\right| = 4$ **Ans.**

Illustration 22 : Find the values of x in the interval $[0,2\pi]$ which satisfy the inequality : $3|2\sin x - 1| \geq 3 + 4\cos^2 x$.

Solution : The given inequality can be written as :

$3|2\sin x - 1| \geq 3 + 4(1 - \sin^2 x) \quad \Rightarrow \quad 3|2\sin x - 1| \geq 7 - 4\sin^2 x$

Let $\sin x = t \Rightarrow 3|2t - 1| \geq 7 - 4t^2$

Case I : For $2t - 1 \geq 0$ i.e. $t \geq 1/2$ we have , $|2t - 1| = (2t - 1)$

$\Rightarrow \quad 3(2t - 1) \geq 7 - 4t^2 \quad \Rightarrow \quad 6t - 3 \geq 7 - 4t^2$

$\Rightarrow \quad 4t^2 + 6t - 10 \geq 0 \quad \Rightarrow \quad 2t^2 + 3t - 5 \geq 0$

$\Rightarrow \quad (t-1)(2t+5) \geq 0 \quad \Rightarrow \quad t \leq -\frac{5}{2}$ and $t \geq 1$

Now for $t \geq \frac{1}{2}$, we get $t \geq 1$ from above conditions i.e. $\sin x \geq 1$

The inequality holds true only for x satisfying the equation $\sin x = 1 \therefore x = \frac{\pi}{2}$ (for $x \in [0,2\pi]$)

Case II : For $2t - 1 < 0 \quad \Rightarrow \quad t < \frac{1}{2}$

we have, $|2t - 1| = -(2t - 1)$

$\Rightarrow \quad -3(2t - 1) \geq 7 - 4t^2 \quad \Rightarrow \quad -6t + 3 \geq 7 - 4t^2$

$\Rightarrow \quad 4t^2 - 6t - 4 \geq 0 \quad \Rightarrow \quad 2t^2 - 3t - 2 \geq 0$

$\Rightarrow \quad (t-2)(2t+1) \geq 0 \quad \Rightarrow \quad t \leq -\frac{1}{2}$ and $t \geq 2$

Again, for $t < \frac{1}{2}$ we get $t \leq -\frac{1}{2}$ from above conditions

i.e. $\sin x \leq -\frac{1}{2} \quad \Rightarrow \quad \frac{7\pi}{6} \leq x \leq \frac{11}{6}\pi$ (for $x \in [0,2\pi]$)

Thus, $x \in \left[\frac{7\pi}{6}, \frac{11\pi}{6}\right] \cup \left\{\frac{\pi}{2}\right\}$ **Ans.**

Illustration 23 : Find the values of θ, for which $\cos 3\theta + \sin 3\theta + (2\sin 2\theta - 3)(\sin\theta - \cos\theta)$ is always positive.

Solution : Given expression can be written as :

$4\cos^3\theta - 3\cos\theta + 3\sin\theta - 4\sin^3\theta + (2\sin 2\theta - 3)(\sin\theta - \cos\theta)$

Applying given condition, we get

$\Rightarrow -4(\sin^3\theta - \cos^3\theta) + 3(\sin\theta - \cos\theta) + (\sin\theta - \cos\theta)(2\sin 2\theta - 3) > 0$

$\Rightarrow -4(\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta) + 3(\sin\theta - \cos\theta) + (\sin\theta - \cos\theta)(2\sin 2\theta - 3) > 0$

$\Rightarrow -4(\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta) + 3(\sin\theta - \cos\theta) + (\sin\theta - \cos\theta)(4\sin\theta\cos\theta - 3) > 0$

$\Rightarrow (\sin\theta - \cos\theta)\{-4 - 4\sin\theta\cos\theta + 3 + 4\sin\theta\cos\theta - 3\} > 0$

$\Rightarrow -4(\sin\theta - \cos\theta) > 0$

$\Rightarrow -4\sqrt{2}\sin\left(\theta - \frac{\pi}{4}\right) > 0 \quad \Rightarrow \sin\left(\theta - \frac{\pi}{4}\right) < 0 \quad \Rightarrow \quad 2n\pi - \pi < \theta - \frac{\pi}{4} < 2n\pi,\ n \in I$

$\Rightarrow 2n\pi - \frac{3\pi}{4} < \theta < 2n\pi + \frac{\pi}{4} \Rightarrow \theta \in \left(2n\pi - \frac{3\pi}{4}, 2n\pi + \frac{\pi}{4}\right),\ n \in I$ **Ans.**

Illustration 24 : The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is - [JEE 98]

(A) 0 (B) 5 (C) 6 (D) 10

Solution : $3\sin^2 x - 7\sin x + 2 = 0$

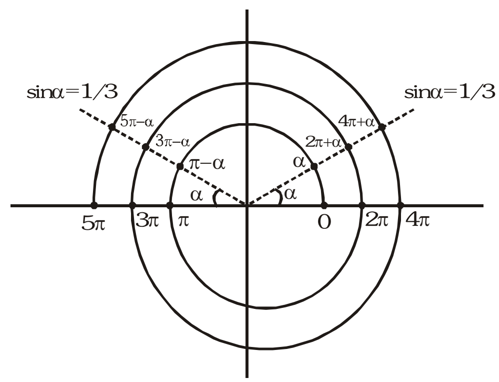
$\Rightarrow \quad (3\sin x - 1)(\sin x - 2) = 0$

$\because \quad \sin x \neq 2$

$\Rightarrow \quad \sin x = \frac{1}{3} = \sin\alpha$ (say)

where α is the least positive value of x

such that $\sin\alpha = \frac{1}{3}$.



Clearly $0 < \alpha < \frac{\pi}{2}$. We get the solution,

$x = \alpha,\ \pi - \alpha,\ 2\pi + \alpha,\ 3\pi - \alpha,\ 4\pi + \alpha$ and $5\pi - \alpha$.

Hence total six values in $[0, 5\pi]$ **Ans.(C)**

ANSWERS FOR DO YOURSELF

1 : (i) (a) $\theta = n\pi + (-1)^n\frac{\pi}{6}, n \in I$ (b) $\theta = (2n+1)\frac{\pi}{3}, n \in I$ (c) $\theta = \frac{4n\pi}{3}, n \in I$

(d) $\theta = \frac{n\pi}{2}, n \in I$ (e) $\theta = n\pi \pm \frac{\pi}{12}, n \in I$ (f) $\theta = 2n\pi + (-1)^{n+1}\pi$, $n \in I$

2 : (i) (a) $x = n\pi + (-1)^{n+1}\frac{\pi}{6}$, $n \in I$ (b) $\alpha = \frac{n\pi}{2}$ or $\alpha = \frac{k\pi}{2} + \frac{3\pi}{8}$, $n, k \in I$

(c) $\theta = n\pi \pm \frac{\pi}{3}, n \in I$ (d) $\theta = n\pi + (-1)^n\alpha$, where $\alpha = \sin^{-1}\left(\frac{\sqrt{17}-1}{8}\right)$ or $\sin^{-1}\left(\frac{-1-\sqrt{17}}{8}\right), n \in I$

(ii) $\theta = \left\{-\frac{\pi}{4}, -\frac{3\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{2}\right\}$

3 : (i) (a) $x = 2n\pi - \frac{\pi}{4}, n \in I$ (b) $2m\pi + \frac{\pi}{2}, m \in I$

4 : (i) $\theta = n\pi$ or $\theta = \frac{m\pi}{3} \pm \frac{\pi}{9}$; $n, m \in I$ (ii) $x = \frac{n\pi}{3}, n \in I$ and $k\pi \pm \frac{\pi}{3}, k \in I$

5 : (i) D (ii) $x = \frac{\pi}{4}$

6 : (i) $\bigcup_{n \in I}\left[2n\pi - \frac{2\pi}{3}, 2n\pi + \frac{2\pi}{3}\right]$ (ii) $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

EXERCISE - 01 CHECK YOUR GRASP

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. The number of solutions of the equation $\frac{\sec x}{1-\cos x}=\frac{1}{1-\cos x}$ in $[0, 2\pi]$ is equal to -

(A) 3 (B) 2 (C) 1 (D) 0

2. The number of solutions of equation $2 + 7\tan^2\theta = 3.25 \sec^2\theta (0 < \theta < 360)$ is -

(A) 2 (B) 4 (C) 6 (D) 8

3. The number of solutions of the equation $\tan^2 x - \sec^{10} x + 1 = 0$ in $(0, 10)$ is -

(A) 3 (B) 6 (C) 10 (D) 11

4. If $(\cos\theta + \cos2\theta)^3 = \cos^3\theta + \cos^3 2\theta$, then the least positive value of θ is equal to -

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

5. The number of solution(s) of $\sin2x + \cos4x = 2$ in the interval $(0, 2\pi)$ is -

(A) 0 (B) 2 (C) 3 (D) 4

6. The complete solution of the equation $7\cos^2 x + \sin x \cos x - 3 = 0$ is given by -

(A) $n\pi+\frac{\pi}{2}; (n \in I)$ (B) $n\pi-\frac{\pi}{4}; (n \in I)$

(C) $n\pi+\tan^{-1}\frac{4}{3}; (n \in I)$ (D) $n\pi+\frac{3\pi}{4}, k\pi+\tan^{-1}\frac{4}{3}; (n, k \in I)$

7. If $\cos(\sin x) = 0$, then x lies in -

(A) $\left(\frac{\pi}{4},\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right)$ (B) $\left(-\frac{\pi}{4},0\right)$ (C) $\left(\pi,\frac{3\pi}{2}\right)$ (D) null set

8. If $0 \le \alpha, \beta \le 90$ and $\tan(\alpha + \beta) = 3$ and $\tan(\alpha - \beta) = 2$ then value of $\sin2\alpha$ is -

(A) $-\frac{1}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2}$ (D) none of these

9. If tanA and tanB are the roots of $x^2 - 2x - 1 = 0$, then $\sin^2(A+B)$ is -

(A) 1 (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2}$ (D) 0

10. If $\cos2x - 3\cos x + 1 = \frac{\operatorname{cosec} x}{\cot x-\cot 2x}$, then which of the following is true ?

(A) $x=(2n+1)\frac{\pi}{2}, n \in I$ (B) $x = 2n\pi, n \in I$

(C) $x=2n\pi\pm\cos^{-1}\left(\frac{2}{5}\right), n \in I$ (D) no real x

11. The solutions of the equation $\sin x + 3\sin2x + \sin3x = \cos x + 3\cos2x + \cos3x$ in the interval $0 \le x \le 2\pi$, are ;

(A) $\frac{\pi}{8},\frac{5\pi}{8},\frac{2\pi}{3}$ (B) $\frac{\pi}{8},\frac{5\pi}{8},\frac{9\pi}{8},\frac{13\pi}{8}$ (C) $\frac{4\pi}{3},\frac{9\pi}{3},\frac{2\pi}{3},\frac{13\pi}{8}$ (D) $\frac{\pi}{8},\frac{5\pi}{8},\frac{9\pi}{3},\frac{4\pi}{3}$

12. If $x \in\left[-\frac{5\pi}{2},\frac{5\pi}{2}\right]$, then the greatest positive solution of $1 + \sin^4 x = \cos^2 3x$ is -

(A) π (B) 2π (C) $\frac{5\pi}{2}$ (D) none of these

13. Number of values of 'x' in $(-2\pi, 2\pi)$ satisfying the equation $2^{\sin^2 x} + 4.2^{\cos^2 x} = 6$ is -

(A) 8 (B) 6 (C) 4 (D) 2

14. General solution for $|\sin x| = \cos x$ is -

(A) $2n\pi + \frac{\pi}{4}$, $n \in I$ (B) $2n\pi \pm \frac{\pi}{4}$, $n \in I$ (C) $n\pi + \frac{\pi}{4}$, $n \in I$ (D) none of these

15. The most general solution of $\tan\theta = -1$, $\cos\theta = \frac{1}{\sqrt{2}}$ is -

(A) $n\pi + \frac{7\pi}{4}, n \in I$ (B) $n\pi + (-1)^n \frac{7\pi}{4}, n \in I$ (C) $2n\pi + \frac{7\pi}{4}, n \in I$ (D) none of these

<u>SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)</u>

16. The solution(s) of the equation $\cos 2x \sin 6x = \cos 3x \sin 5x$ in the interval $[0, \pi]$ is/are -

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{2}$ (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

17. The equation $4\sin^2 x - 2(\sqrt{3}+1)\sin x + \sqrt{3} = 0$ has -

(A) 2 solutions in $(0, \pi)$ (B) 4 solutions in $(0, 2\pi)$ (C) 2 solutions in $(-\pi, \pi)$ (D) 4 solutions in $(-\pi, \pi)$

18. If $\cos^2 2x + 2\cos^2 x = 1$, $x \in (-\pi, \pi)$, then x can take the values -

(A) $\pm\frac{\pi}{2}$ (B) $\pm\frac{\pi}{4}$ (C) $\pm\frac{3\pi}{4}$ (D) none of these

19. The solution(s) of the equation $\sin 7x + \cos 2x = -2$ is/are -

(A) $x = \frac{2k\pi}{7} + \frac{3\pi}{14}, k \in I$ (B) $x = n\pi + \frac{\pi}{4}, n \in I$ (C) $x = 2n\pi + \frac{\pi}{2}, n \in I$ (D) none of these

20. Set of values of x in $(-\pi, \pi)$ for which $|4\sin x - 1| < \sqrt{5}$ is given by -

(A) $\left(\frac{\pi}{10}, \frac{3\pi}{10}\right)$ (B) $\left(-\frac{\pi}{10}, \frac{3\pi}{10}\right)$ (C) $\left(\frac{\pi}{10}, -\frac{3\pi}{10}\right)$ (D) $\left(-\frac{\pi}{10}, -\frac{3\pi}{10}\right)$

CHECK YOUR GRASP — **ANSWER KEY** — EXERCISE-1

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	B	A	B	A	D	D	B	C	D
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B	B	C	B	C	A,B,D	B,D	A,B,C	C	B

EXERCISE - 02

BRAIN TEASERS

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. If $\cos^2x + \cos^2 2x + \cos^2 3x = 1$ then -

(A) $x = (2n + 1)\frac{\pi}{4}$, $n \in I$ (B) $x = (2n + 1)\frac{\pi}{2}$, $n \in I$ (C) $x = n\pi \pm \frac{\pi}{6}$, $n \in I$ (D) none of these

2. If $4\cos^2\theta + \sqrt{3} = 2(\sqrt{3} + 1)\cos\theta$, then θ is -

(A) $2n\pi \pm \frac{\pi}{3}$, $n \in I$ (B) $2n\pi \pm \frac{\pi}{4}$, $n \in I$ (C) $2n\pi \pm \frac{\pi}{6}$, $n \in I$ (D) none of these

3. Set of values of 'α' in $[0, 2\pi]$ for which $m = \log_{\left(x+\frac{1}{x}\right)}(2\sin\alpha - 1) \le 0$, is -

(A) $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$ (B) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (C) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \pi\right)$ (D) $\left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$

4. If $(a + 2)\sin\alpha + (2a - 1)\cos\alpha = (2a + 1)$, then $\tan\alpha$ =

(A) 3/4 (B) 4/3 (C) $\frac{2a}{a^2+1}$ (D) $\frac{2a}{a^2-1}$

5. If $\theta_1, \theta_2, \theta_3, \theta_4$ are the roots of the equation $\sin(\theta + \alpha) = k\sin 2\theta$, no two of which differ by a multiple of 2π, then $\theta_1 + \theta_2 + \theta_3 + \theta_4$ is equal to -

(A) $2n\pi$, $n \in Z$ (B) $(2n + 1)\pi$, $n \in Z$ (C) $n\pi$, $n \in Z$ (D) none of these

6. The number of solution(s) of the equation $\cos 2\theta = \left(\sqrt{2}+1\right)\left(\cos\theta - \frac{1}{\sqrt{2}}\right)$, in the interval $\left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$, is -

(A) 4 (B) 1 (C) 2 (D) 3

7. The value(s) of θ lying between 0 & 2π satisfying the equation : $r\sin\theta = \sqrt{3}$ & $r + 4\sin\theta = 2(\sqrt{3} + 1)$ is/are -

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

8. The value(s) of θ, which satisfy $3 - 2\cos\theta - 4\sin\theta - \cos 2\theta + \sin 2\theta = 0$ is/are -

(A) $\theta = 2n\pi$; $n \in I$ (B) $2n\pi + \frac{\pi}{2}$; $n \in I$ (C) $2n\pi - \frac{\pi}{2}$; $n \in I$ (D) $n\pi$; $n \in I$

9. Given that A, B are positive acute angles and $\sqrt{3}\sin 2A = \sin 2B$ & $\sqrt{3}\sin^2 A + \sin^2 B = \frac{\sqrt{3}-1}{2}$, then A or B may take the value(s) -

(A) 15 (B) 30 (C) 45 (D) 75

10. The solution(s) of $4\cos^2x\sin x - 2\sin^2x = 3\sin x$ is/are -

(A) $n\pi$; $n \in I$ (B) $n\pi + (-1)^n\frac{\pi}{10}$; $n \in I$

(C) $n\pi + (-1)^n\left(-\frac{3\pi}{10}\right)$; $n \in I$ (D) none of these

11. If $\left(\frac{1-a\sin x}{1+a\sin x}\right)\sqrt{\frac{1+2a\sin x}{1-2a\sin x}} = 1$, where $a \in R$ then -

(A) $x \in \phi$ (B) $x \in R\ \forall\ a$

(C) $a = 0$, $x \in R$ (D) $a \in R$, $x \in n\pi$, where $n \in I$

12. The general solution of the following equation : $2(\sin x - \cos 2x) - \sin 2x(1 + 2\sin x) + 2\cos x = 0$ is/are -

(A) $x = 2n\pi$; $n \in I$

(B) $n\pi + (-1)^n\left(-\frac{\pi}{2}\right)$; $n \in I$

(C) $x = n\pi + (-1)^n\frac{\pi}{6}$; $n \in I$

(D) $x = n\pi + (-1)^n\frac{\pi}{4}$; $n \in I$

13. The value(s) of θ, which satisfy the equation : $2\cos^3 3\theta + 3\cos 3\theta + 4 = 3\sin^2 3\theta$ is/are -

(A) $\frac{2n\pi}{3}+\frac{2\pi}{9}, n \in I$

(B) $\frac{2n\pi}{3}-\frac{2\pi}{9}, n \in I$

(C) $\frac{2n\pi}{5}+\frac{2\pi}{5}, n \in I$

(D) $\frac{2n\pi}{5}-\frac{2\pi}{5}, n \in I$

14. If $x \neq \frac{k\pi}{2}$, $k \in I$ and $(\cos x)^{\sin^2 x - 4\sin x + 3} = 1$, then all solutions of x are given by -

(A) $n\pi + (-1)^n\frac{\pi}{2}$; $n \in I$

(B) $2n\pi \pm \frac{\pi}{2}$; $n \in I$

(C) $(2n+1)\pi - \frac{\pi}{2}$; $n \in I$

(D) none of these

15. Using four values of θ satisfying the equation $8\cos^4\theta + 15\cos^2\theta - 2 = 0$ in the interval $(0,4\pi)$, an arithmetic progression is formed, then :

(A) The common difference of A.P. may be π.

(B) The common difference of A.P. may be 2π.

(C) Two such different A.P. can be formed.

(D) Four such different A.P. can be formed.

BRAIN TEASERS — ANSWER KEY — EXERCISE-2

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A,B,C	A,C	B	B,D	B	C	A,B,C,D	A,B	A,B	A,B,C
Que.	11	12	13	14	15					
Ans.	C,D	A,B,C	A,B	D	A,D					

EXERCISE - 03

MISCELLANEOUS TYPE QUESTIONS

TRUE / FALSE

1. For all θ in $\left[0, \frac{\pi}{2}\right]$, $\cos(\sin\theta) > \sin(\cos\theta)$.
2. Number of solutions of the equation $\cos(x^2) = 2^{|x|}$ is two.

FILL IN THE BLANKS

1. Number of values of θ in $[0, 2\pi]$ for which vectors $\vec{v}_1 = (2\cos\theta)\tilde{i} - (\cos\theta)\tilde{j} + \tilde{k}$ and $\vec{v}_2 = (\cos\theta)\tilde{i} + 5\tilde{j} + 2\tilde{k}$ are perpendicular is ....
2. The solution set of the system of equations, $x + y = \frac{2\pi}{3}$, $\cos x + \cos y = \frac{3}{2}$, where x & y are real, is ....
3. If $\operatorname{cosec}\theta + \cot\theta = \frac{1}{2}$, then θ lies in .... quadrant.
4. Number of solutions of the equation $\sin 5\theta \cos 3\theta = \sin 9\theta \cos 7\theta$ in $\left[0, \frac{\pi}{4}\right]$ is ....

MATCH THE COLUMN

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE** statement in **Column-II**.

1. On the left, equation with interval is given and on the right number of solutions are given, match the column.

	Column-I		Column-II
(A)	$n\|\sin x\| = m\ \|\cos x\|$ in $[0, 2\pi]$ where n > m and are positive integers	(p)	2
(B)	$\sum_{r=1}^{5} \cos rx = 5$ in $[0, 2\pi]$	(q)	4
(C)	$2^{1+\|\cos x\|+\|\cos x\|^2\ldots\ldots\infty} = 4$ in $(-\pi, \pi)$	(r)	3
(D)	$\tan\theta + \tan 2\theta + \tan 3\theta = \tan\theta \tan 2\theta \tan 3\theta$ in $(0, \pi)$	(s)	1

ASSERTION & REASON

These questions contains, Statement-I (assertion) and Statement-II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for Statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

1. **Statement-I** : For any real value of $\theta \neq (2n+1)\pi$ or $(2n+1)\pi/2$, $n \in I$, the value of the expression $y = \frac{\cos^2\theta - 1}{\cos^2\theta + \cos\theta}$ is $y \le 0$ or $y \ge 2$ (either less than or equal to zero or greater than or equal to two)
   Because
   Statement-II : $\sec\theta \in (-\infty, -1] \cup [1, \infty)$ for all real values of θ.
   (A) A (B) B (C) C (D) D
2. **Statement-I** : The equation $\sqrt{3}\cos x - \sin x = 2$ has exactly one solution in $[0, 2\pi]$.
   Because
   Statement-II : For equations of type $a\cos\theta + b\sin\theta = c$ to have real solutions in $[0, 2\pi]$, $|c| \le \sqrt{a^2 + b^2}$ should hold true.
   (A) A (B) B (C) C (D) D

COMPREHENSION BASED QUESTIONS

Comprehension # 1 :

Let S_1 be the set of all those solutions of the equation $(1 + a)\cos\theta \cos(2\theta - b) = (1 + a \cos 2\theta) \cos(\theta - b)$ which are independent of a and b and S_2 be the set of all such solutions which are dependent on a and b.

On the basis of above information, answer the following questions :

1. The sets S_1 and S_2 are given by -

(A) $\{n\pi, n \in Z\}$ and $\{m\pi + (-1)^m \sin^{-1}(a \sin b), m \in Z\}$

(B) $\left\{\frac{n\pi}{2}, n \in Z\right\}$ and $\{m\pi + (-1)^m \sin^{-1}(a \sin b), m \in Z\}$

(C) $\left\{\frac{n\pi}{2}, n \in Z\right\}$ and $\{m\pi + (-1)^m \sin^{-1}((a/2)\sin b), m \in Z\}$

(D) none of these

2. Condition that should be imposed on a and b such that S_2 is non-empty -

(A) $\left|\frac{a}{2}\sin b\right| < 1$ (B) $\left|\frac{a}{2}\sin b\right| \le 1$ (C) $|a \sin b| \le 1$ (D) none of these

3. All the permissible values of b, if a = 0 and S_2 is a subset of $(0, \pi)$ is -

(A) $b \in (-n\pi, 2n\pi)$; $n \in Z$ (B) $b \in (-n\pi, 2\pi - n\pi)$; $n \in Z$

(C) $b \in (-n\pi, n\pi)$; $n \in Z$ (D) none of these

MISCELLANEOUS TYPE QUESTION **ANSWER KEY** EXERCISE -3

- ***True / False***
  1. T **2.** F
- ***Fill in the Blanks***
  1. 2 **2.** ϕ **3.** II quadrant **4.** 5
- ***Match the Column***
  1. (A) $\to$ (q), (B) $\to$ (p), (C) $\to$ (q), (D) $\to$ (p)
- ***Assertion & Reason***
  1. D **2.** B
- ***Comprehension Based Quesions***
  Comprehension #1 : 1. D **2.** C **3.** B

EXERCISE - 04 [A] CONCEPTUAL SUBJECTIVE EXERCISE

1. If sinA = sinB & cosA = cosB, find the values of A in terms of B.

2. Solve the equation : $1 + 2\operatorname{cosec} x = -\dfrac{\sec^2 \frac{x}{2}}{2}$.

3. Solve the equation : $\dfrac{\sqrt{3}}{2}\sin x - \cos x = \cos^2 x$.

4. Solve the equation : $\cot x - 2\sin 2x = 1$.

5. If α & β satisfy the equation, $a\cos 2\theta + b\sin 2\theta = c$ then prove that : $\cos^2\alpha + \cos^2\beta = \dfrac{a^2 + ac + b^2}{a^2 + b^2}$.

6. Solve for x, $\sqrt{13 - 18\tan x} = 6\tan x - 3$, where $-2\pi < x < 2\pi$.

7. Find all the values of θ satisfying the equation : $\sin\theta + \sin 5\theta = \sin 3\theta$ such that $0 \le \theta \le \pi$.

8. Solve : $\cot\theta + \operatorname{cosec}\theta = \sqrt{3}$ for values of θ between 0 & 360 .

9. Solve : $\sin 5x = \cos 2x$ for all values of x between 0 & 180 .

10. Solve the equation : $(1 - \tan\theta)(1 + \sin 2\theta) = 1 + \tan\theta$.

11. Find the general solution of $\sec 4\theta - \sec 2\theta = 2$.

12. Solve the equation : $\cos 3x.\cos^3 x + \sin 3x.\sin^3 x = 0$.

13. Solve for x : $\sin 3\alpha = 4\sin\alpha\sin(x + \alpha)\sin(x - \alpha)$ where α is a constant $\ne n\pi$, $n \in I$.

14. Solve the inequality : $\sin 3x < \sin x$.

15. Solve the inequality : $\tan^2 x - (\sqrt{3} + 1)\tan x + \sqrt{3} < 0$.

16. Find the smallest positive value of x and y satisfying the equations : $x - y = \dfrac{\pi}{4}$ & $\cot x + \cot y = 2$.

17. Find the value(s) of k for which the equation $\sin x + \cos(k + x) + \cos(k - x) = 2$ has real solutions.

18. Solve : $\tan\theta + \tan\left(\theta + \dfrac{\pi}{3}\right) + \tan\left(\theta + \dfrac{2\pi}{3}\right) = 3$.

19. Solve : $\sin 2\theta = \cos 3\theta$, $0 \le \theta \le 360$.

20. Find all values of θ satisfying the equation $\sin 7\theta = \sin\theta + \sin 3\theta$, where $0 \le \theta \le \pi$.

CONCEPTUAL SUBJECTIVE EXERCISE — ANSWER KEY — EXERCISE-4(A)

1. $A = 2n\pi + B$, $n \in I$

2. $x = 2n\pi - \dfrac{\pi}{2}$, $n \in I$

3. $x = 2n\pi \pm \pi$ or $2n\pi + \dfrac{\pi}{3}$, $n \in I$

4. $x = \dfrac{\pi}{8} + \dfrac{K\pi}{2}$ or $x = \dfrac{3\pi}{4} + K\pi$, $K \in I$

6. $\alpha - 2\pi$; $\alpha - \pi$, α, $\alpha + \pi$, where $\alpha = \tan^{-1}\dfrac{2}{3}$

7. $0, \dfrac{\pi}{6}, \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{5\pi}{6}$ & π

8. $\theta = 60$

9. $\dfrac{90°}{7}$, 30 , $\dfrac{450°}{7}$, $\dfrac{810°}{7}$, 150 , $\dfrac{1170°}{7}$

10. $n\pi$ or $\left(n\pi - \dfrac{\pi}{4}\right)$, $n \in I$

11. $\theta = \dfrac{2n\pi}{5} \pm \dfrac{\pi}{10}$ or $2n\pi \pm \dfrac{\pi}{2}$, $n \in I$

12. $(2n+1)\dfrac{\pi}{4}$, $n \in I$

13. $n\pi \pm \dfrac{\pi}{3}$, $n \in I$

14. $x \in \left(2n\pi + \dfrac{\pi}{4}, 2n\pi + \dfrac{3\pi}{4}\right) \cup \left(2n\pi - \dfrac{\pi}{4}, 2n\pi\right) \cup \left(2n\pi + \pi, 2n\pi + \dfrac{5\pi}{4}\right)$, $n \in I$

15. $n\pi + \dfrac{\pi}{4} < x < n\pi + \dfrac{\pi}{3}$, $n \in I$

16. $x = \dfrac{5\pi}{12}$, $y = \dfrac{\pi}{6}$

17. $n\pi - \dfrac{\pi}{6} \le k \le n\pi + \dfrac{\pi}{6}$, $n \in I$

18. $\theta = (4n+1)\dfrac{\pi}{12}$; $n \in I$

19. θ = 18 , 90 , 162 , 234 , 270 , 306

20. $0, \dfrac{\pi}{12}, \dfrac{\pi}{3}, \dfrac{5\pi}{12}, \dfrac{7\pi}{12}, \dfrac{2\pi}{3}, \dfrac{11\pi}{12}, \pi$

EXERCISE - 04 [B]

BRAIN STORMING SUBJECTIVE EXERCISE

1. Find all values of θ, between 0 & π, which satisfy the equation $\cos\theta\cos2\theta\cos3\theta = 1/4$.

2. Find the general solution of the trigonometric equation :

$$\sqrt{16\cos^4 x - 8\cos^2 x + 1} + \sqrt{16\cos^4 x - 24\cos^2 x + 9} = 2.$$

3. Find the principal solution of the trigonometric equation :

$$\sqrt{\cot 3x + \sin^2 x - \frac{1}{4}} + \sqrt{\sqrt{3}\cos x + \sin x - 2} = \sin\frac{3x}{2} - \frac{\sqrt{2}}{2}.$$

4. Solve : $2\sin\left(3x + \frac{\pi}{4}\right) = \sqrt{1 + 8\sin 2x.\cos^2 2x}$.

5. Solve for x, $(-\pi \le x \le \pi)$ the equation : $2(\cos x + \cos 2x) + \sin 2x(1 + 2\cos x) = 2\sin x$.

6. Solve : $\log_{\frac{-x^2-6x}{10}}(\sin 3x + \sin x) = \log_{\frac{-x^2-6x}{10}}(\sin 2x)$.

7. Find the set of values of 'a' for which the equation, $\sin^4 x + \cos^4 x + \sin 2x + a = 0$ possesses solutions. Also find the general solution for these values of 'a'.

8. Solve : $\cos(\pi . 3^x) - 2\cos^2(\pi . 3^x) + 2\cos(4\pi . 3^x) - \cos(7\pi . 3^x)$

$= \sin(\pi . 3^x) + 2\sin^2(\pi . 3^x) - 2\sin(4\pi . 3^x) + 2\sin(\pi . 3^{x+1}) - \sin(7\pi . 3^x)$

9. Find the least positive angle measured in degrees satisfying the equation :

$\sin^3 x + \sin^3 2x + \sin^3 3x = (\sin x + \sin 2x + \sin 3x)^3$.

10. Solve for x, y : $\begin{cases} \sin x \cos y = \dfrac{1}{4} \\ 3\tan x = \tan y \end{cases}$.

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BRAIN STORMING SUBJECTIVE EXERCISE **ANSWER KEY** EXERCISE-4(B)

1. $\frac{\pi}{8}, \frac{\pi}{3}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{2\pi}{3}, \frac{7\pi}{8}$ 2. $x \in \left[n\pi + \frac{\pi}{6}, n\pi + \frac{\pi}{3}\right] \cup \left[n\pi + \frac{2\pi}{3}, n\pi + \frac{5\pi}{6}\right]$, $n \in I$

3. $x = \pi/6$ only 4. $x = 2n\pi + \frac{\pi}{12}$ or $2n\pi + \frac{17\pi}{12}$; $n \in I$ 5. $\left\{-\pi, -\frac{\pi}{2}, -\frac{\pi}{3}, \frac{\pi}{3}, \pi\right\}$

6. $x = -\frac{5\pi}{3}$ 7. $\frac{1}{2}\left[n\pi + (-1)^n \sin^{-1}\left(1 - \sqrt{2a+3}\right)\right]$ where $n \in I$ and $a \in \left[-\frac{3}{2}, \frac{1}{2}\right]$

8. $x = \log_3\left(\frac{2k}{3} - \frac{1}{6}\right)$, $k \in N$; $x = \log_3\left(\frac{n}{2}\right)$, $n \in N$; $x = \log_3\left(\frac{1}{8} + \frac{m}{2}\right)$, $m \in N \cup \{0\}$

9. 72 10. $\begin{cases} x = (4k+1)\frac{\pi}{4} + \frac{n\pi}{2} + (-1)^{n+1}\frac{\pi}{12} \\ y = (4k+1)\frac{\pi}{4} - \frac{n\pi}{2} - (-1)^{n+1}\frac{\pi}{12} \end{cases}$, $n \in I$

EXERCISE - 05 [A]

JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

1. Find the no. of roots of the equation tan x + sec x = 2 cos x in the interval $[0,2\pi]$ - [AIEEE 2002, IIT 1993]
(1) 1 (2) 2 (3) 3 (4) 4

2. General solution of tan 5θ = cot 2θ is- [AIEEE 2002]
(1) $\theta = \frac{n\pi}{7} + \frac{\pi}{14}$ (2) $\theta = \frac{n\pi}{7} + \frac{\pi}{5}$ (3) $\theta = \frac{n\pi}{7} + \frac{\pi}{2}$ (4) $\theta = \frac{n\pi}{7} + \frac{\pi}{3}, n \in Z$

3. The number of values of x in the interval $[0,3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is- [AIEEE 2006]
(1) 6 (2) 1 (3) 2 (4) 4

4. If $0 < x < \pi$, and $\cos x + \sin x = \frac{1}{2}$, then tan x is - [AIEEE 2006]
(1) $(4-\sqrt{7})/3$ (2) $-(4+\sqrt{7})/3$ (3) $(1+\sqrt{7})/4$ (4) $(1-\sqrt{7})/4$

5. Let A and B denote the statements

$$A : \cos\alpha + \cos\beta + \cos\gamma = 0$$
$$B : \sin\alpha + \sin\beta + \sin\gamma = 0$$

If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$, then :- [AIEEE 2009]
(1) Both **A** and **B** are true (2) Both **A** and **B** are false
(3) **A** is true and **B** is false (4) **A** is false and **B** is true

6. The possible values of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are: [AIEEE 2011]
(1) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$ (2) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$
(3) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$ (4) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

PREVIOUS YEARS QUESTIONS — ANSWER KEY — EXERCISE-5 [A]

Que.	1	2	3	4	5	6				
Ans.	2	1	4	2	1	1				

EXERCISE - 05 [B]

JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS

1. The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution is

(A) 4 (B) 8 (C) 10 (D) 12

[JEE 2002 (Screening), 3]

2. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$, where $\alpha, \beta \in [-\pi, \pi]$, numbers of pairs of α, β which satisfy both the equations is

(A) 0 (B) 1 (C) 2 (D) 4

[JEE 2005 (Screening)]

3. If $0 < \theta < 2\pi$, then the intervals of values of θ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, is

(A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (B) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$ (C) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (D) $\left(\frac{41\pi}{48}, \pi\right)$

[JEE 2006, 3]

4. The number of solutions of the pair of equations

$2\sin^2\theta - \cos 2\theta = 0$

$2\cos^2\theta - 3\sin\theta = 0$

in the interval $[0, 2\pi]$ is [JEE 2007, 3]

(A) zero (B) one (C) two (D) four

5. The number of values of θ in the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$, is [JEE 2010, 3]

6. The positive integer value of $n > 3$ satisfying the equation

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$$ is [JEE 2011, 4]

7. Let $\theta, \varphi \in [0, 2\pi]$ be such that

$$2\cos\theta(1 - \sin\varphi) = \sin^2\theta\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\cos\varphi - 1, \quad \tan(2\pi - \theta) > 0 \quad \text{and} \quad -1 < \sin\theta < -\frac{\sqrt{3}}{2}.$$

Then φ **cannot** satisfy- [JEE 2012, 4]

(A) $0 < \varphi < \frac{\pi}{2}$ (B) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$ (C) $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$ (D) $\frac{3\pi}{2} < \varphi < 2\pi$

PREVIOUS YEARS QUESTIONS	ANSWER KEY	EXERCISE-5 [B]
1. B **2.** D **3.** A **4.** C **5.** 3 **6.** 7 **7.** A,C,D		